On the Traveling Salesperson Problem with Simple Temporal Constraints

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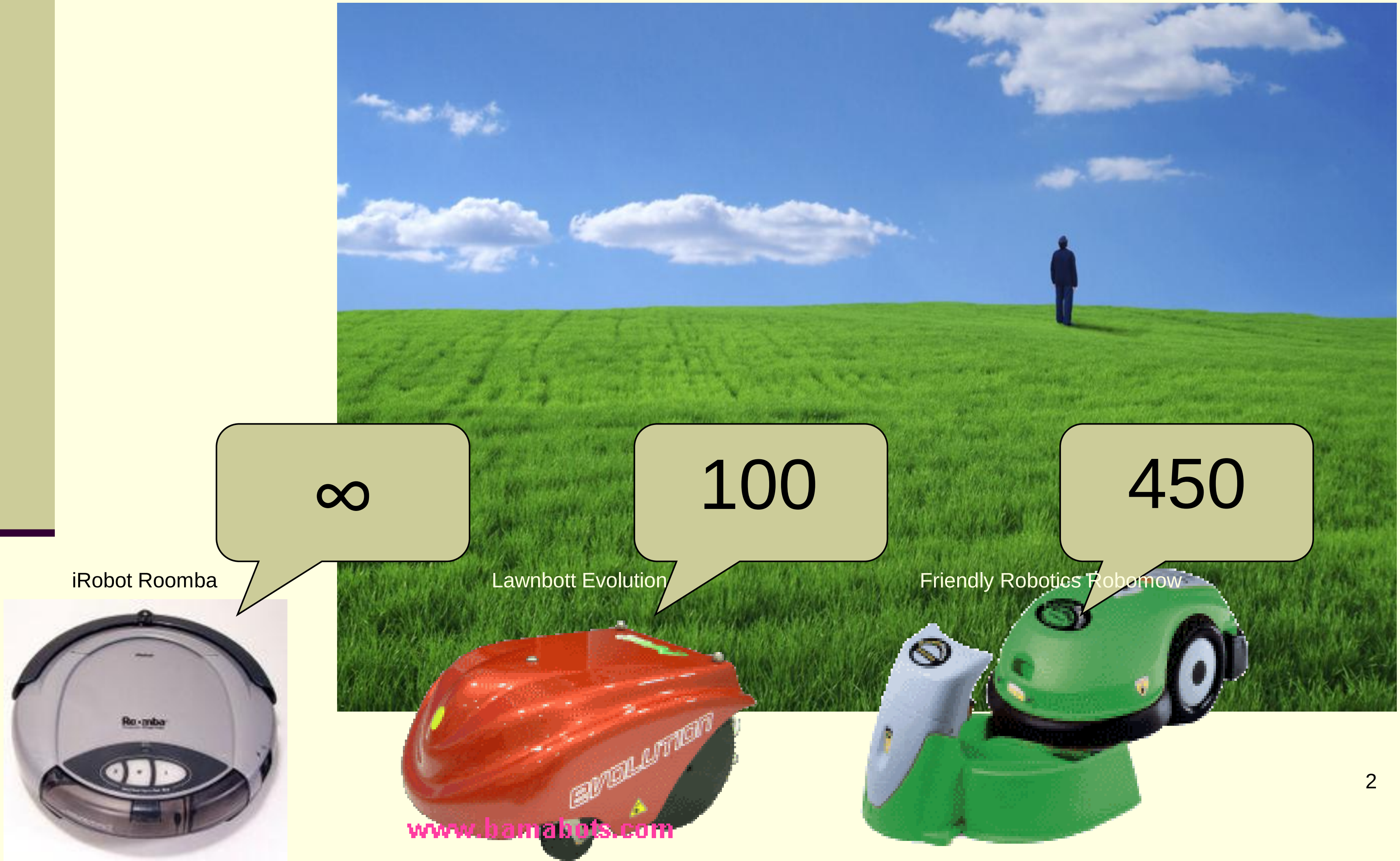
Auction-Based Robot Coordination

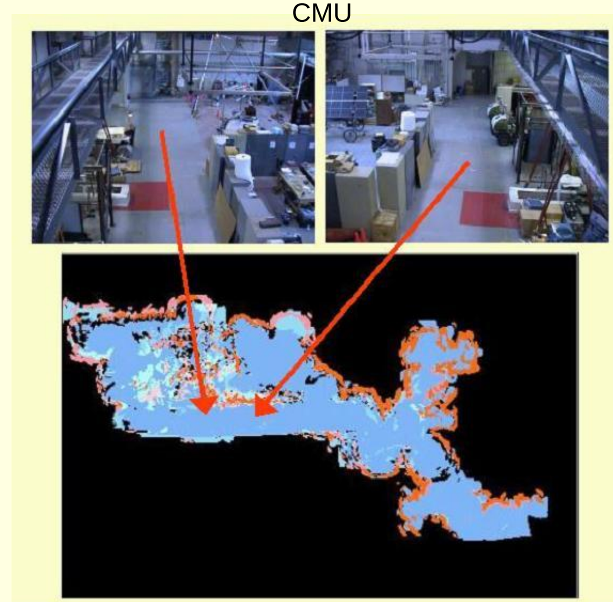


Auction-Based Robot Coordination

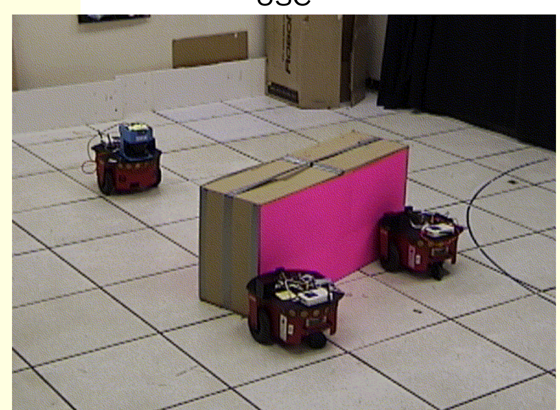
Examples:

- TraderBots (CMU)
- Murdoch (USC)
- COMSTAR (U of Nebraska)
- CBAA (MIT)
- DEMiR-CF (Georgia Tech)
- U of Minnesota

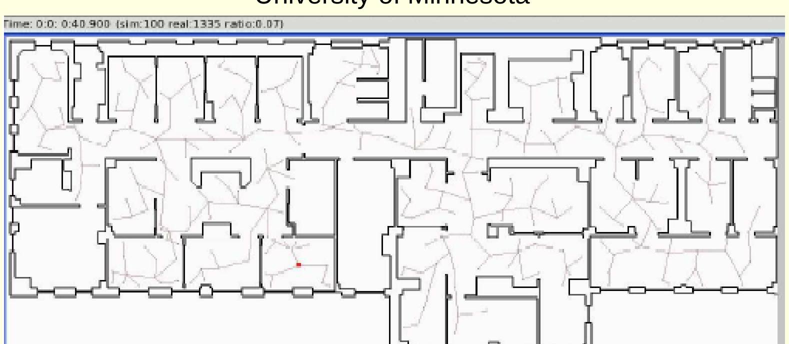
CMU



USC



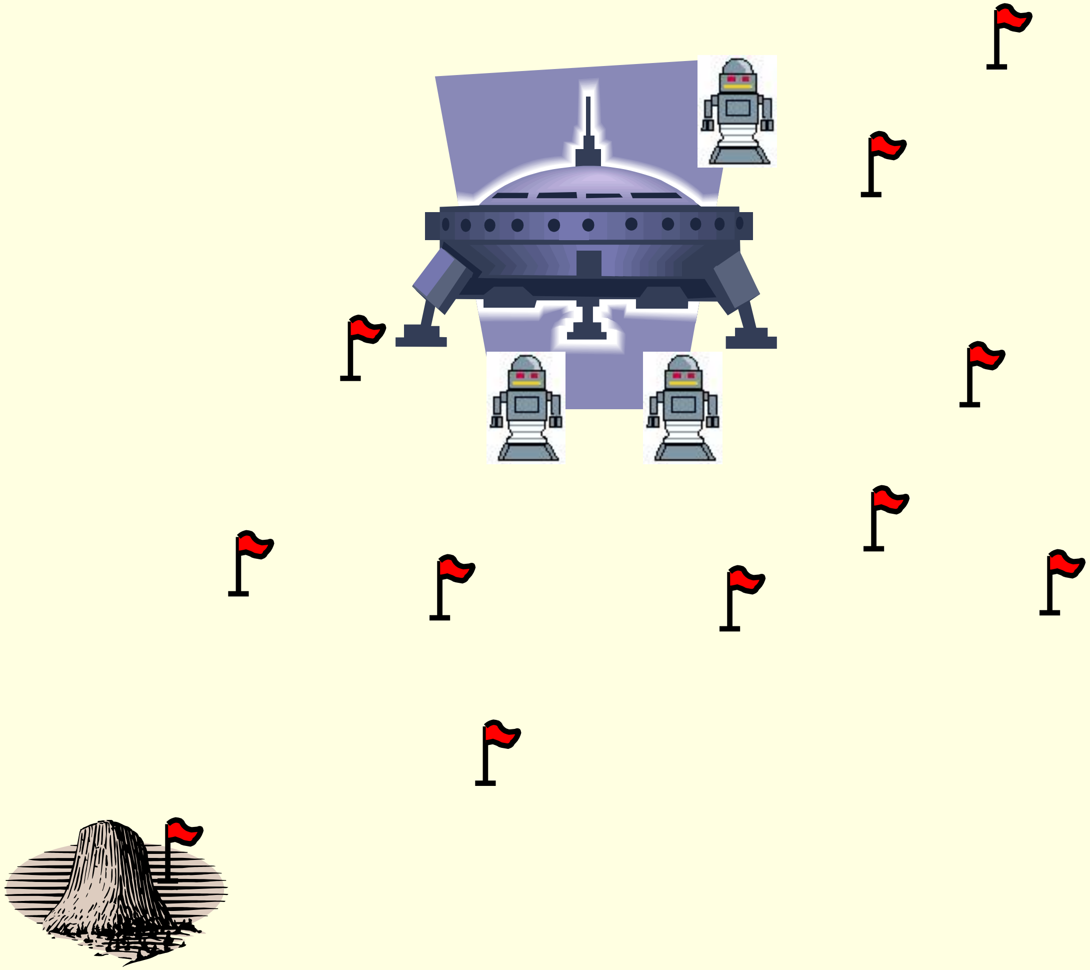
University of Minnesota



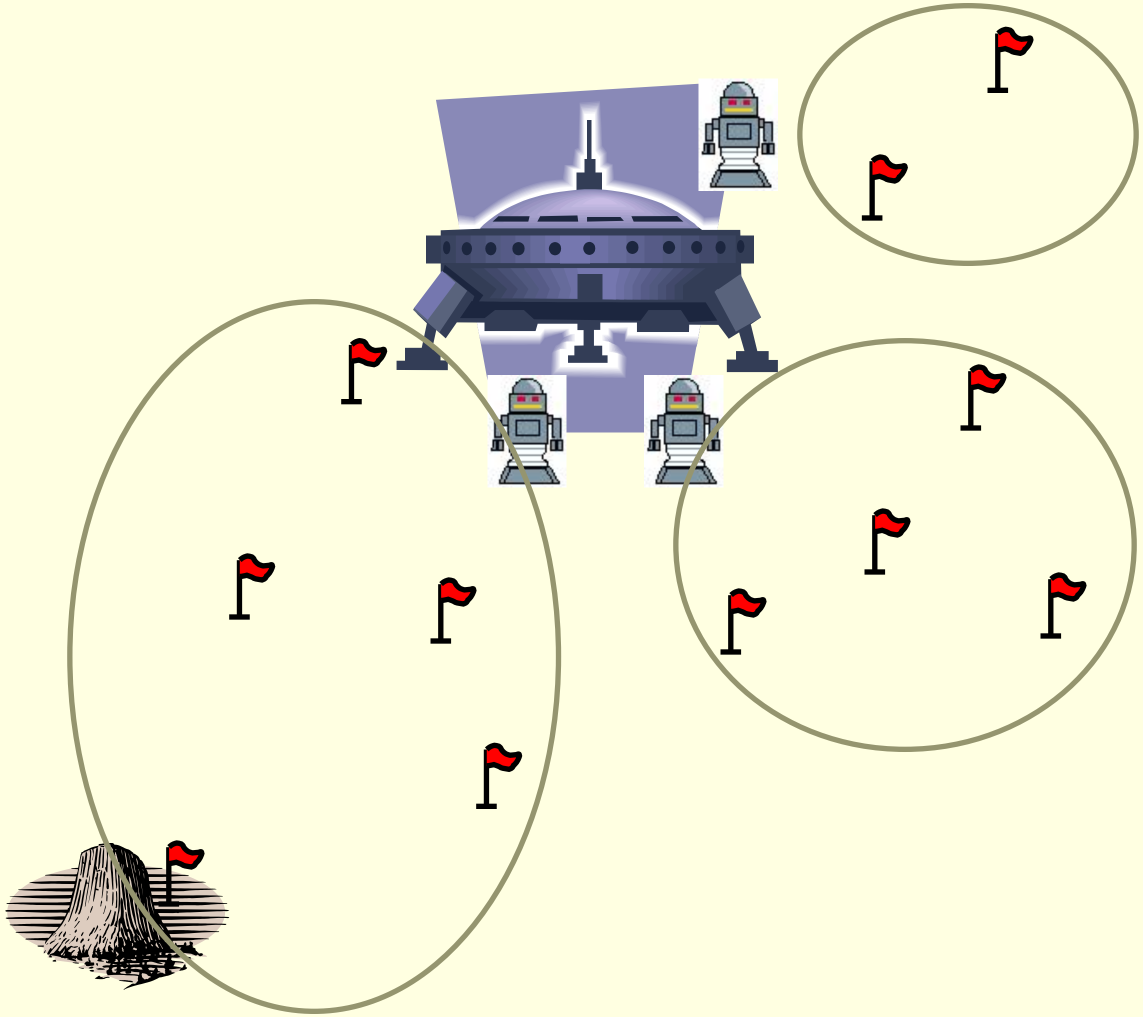
Task: Multi-Robot Routing

- A team of robots has to visit given targets spread out over some known or unknown terrain so that each target is visited by one of the robots
- Examples:
  - Planetary surface exploration
  - Facility surveillance
  - Search and rescue

Task: Multi-Robot Routing



Task: Multi-Robot Routing



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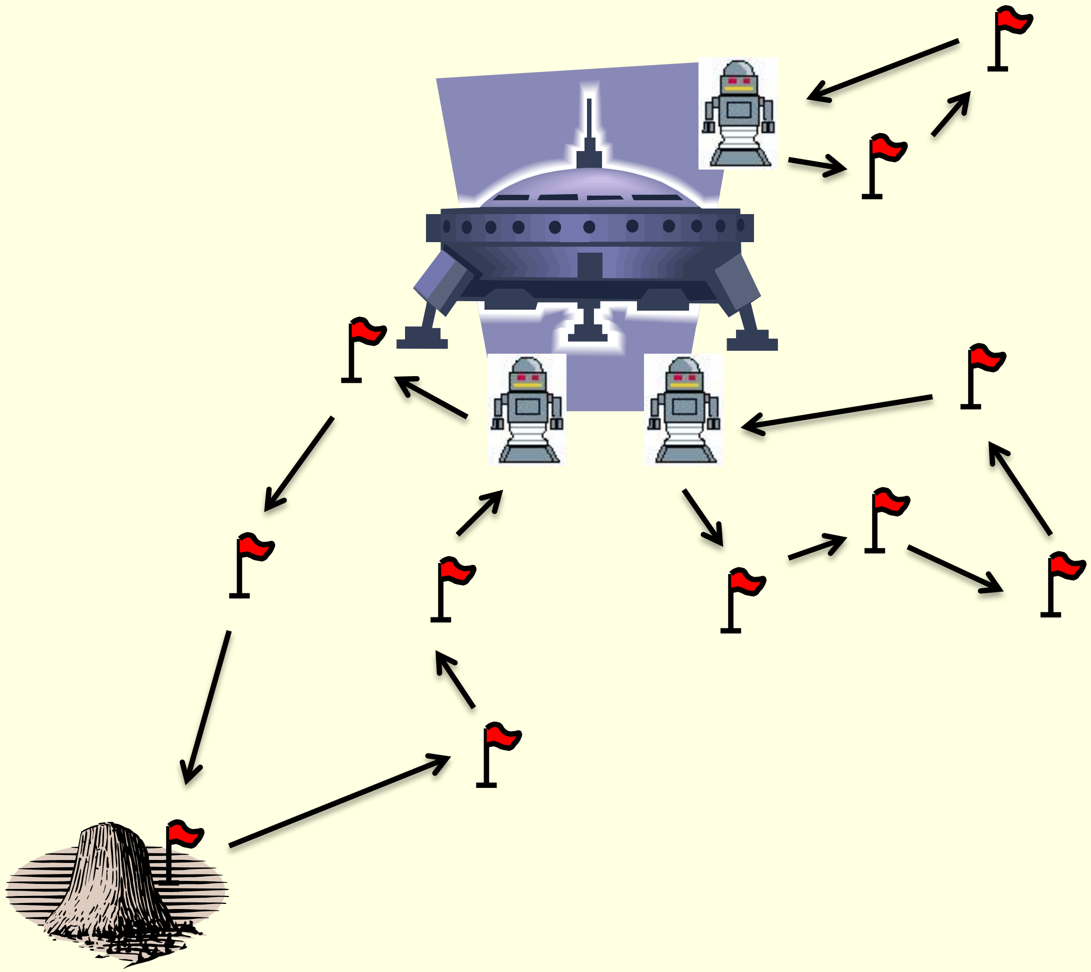
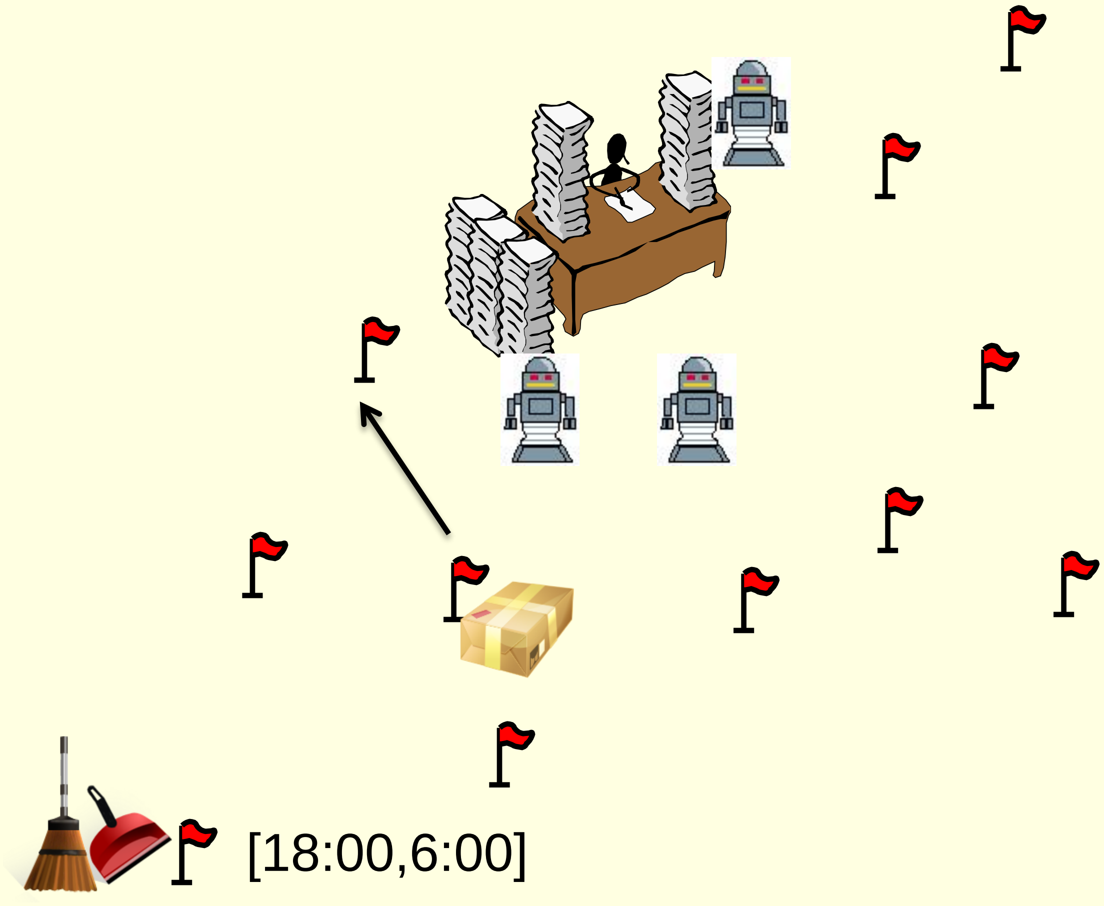


Task: Multi-Robot Routing



Related Work

- Robotics
  - e.g. multi-robot routing with time windows
- Theoretical Computer Science
  - e.g. prize-collecting TSP with time windows
- Operations Research
  - e.g. time-constrained TSPs
- Artificial Intelligence Planning
  - e.g. temporal planning

Motivation

Spatial Reasoning

- Traveling Salesperson Problem (TSP)

Temporal Reasoning

- Simple Temporal Problem (STP)

Traveling Salesperson Problem
with Simple Temporal Constraints
(TSP-STC)

study the time complexity of solving TSP-STCs

Spatial Reasoning: TSP

- Traveling Salesperson Problem (TSP): optimization problem
  - V: locations
  - d: travel distances between locations
- Decide in which order to visit the locations
  to minimize the travel distance
  and possibly satisfy given spatial constraints
  - for example,
    start location = end location
- Time Complexity: NP-hard

Spatial Reasoning: TSP

- TSP
  - symmetric travel distances (= undirected edges)
- ATSP
  - asymmetric travel distances (= directed edges)

Temporal Reasoning: STP

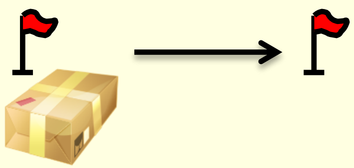
- Simple Temporal Problem (STP): satisfaction problem
  - X: tasks
  - E: simple temporal constraints (STC) between tasks

- Decide when to execute the tasks to satisfy the given STCs

Temporal Reasoning: STP

- Expressivity
  - Task a before Task b

$$a \xrightarrow{[\varepsilon,\infty]} b$$



  - Task a during time window [3,8]

$$0 \xrightarrow{[3,8]} c$$

- Time Complexity: Polynomial

TSP-STC: Definition

Spatial Reasoning

- Traveling Salesperson Problem (TSP)

Temporal Reasoning

- Simple Temporal Problem (STP)

Traveling Salesperson Problem with Simple Temporal Constraints (TSP-STC)

study the time complexity of solving TSP-STCs

TSP-STC: Definition

- TSP-STC
  - TSP: optimization (objective function)
    - V: locations
    - d: travel distances between locations

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TSP-STC: Definition

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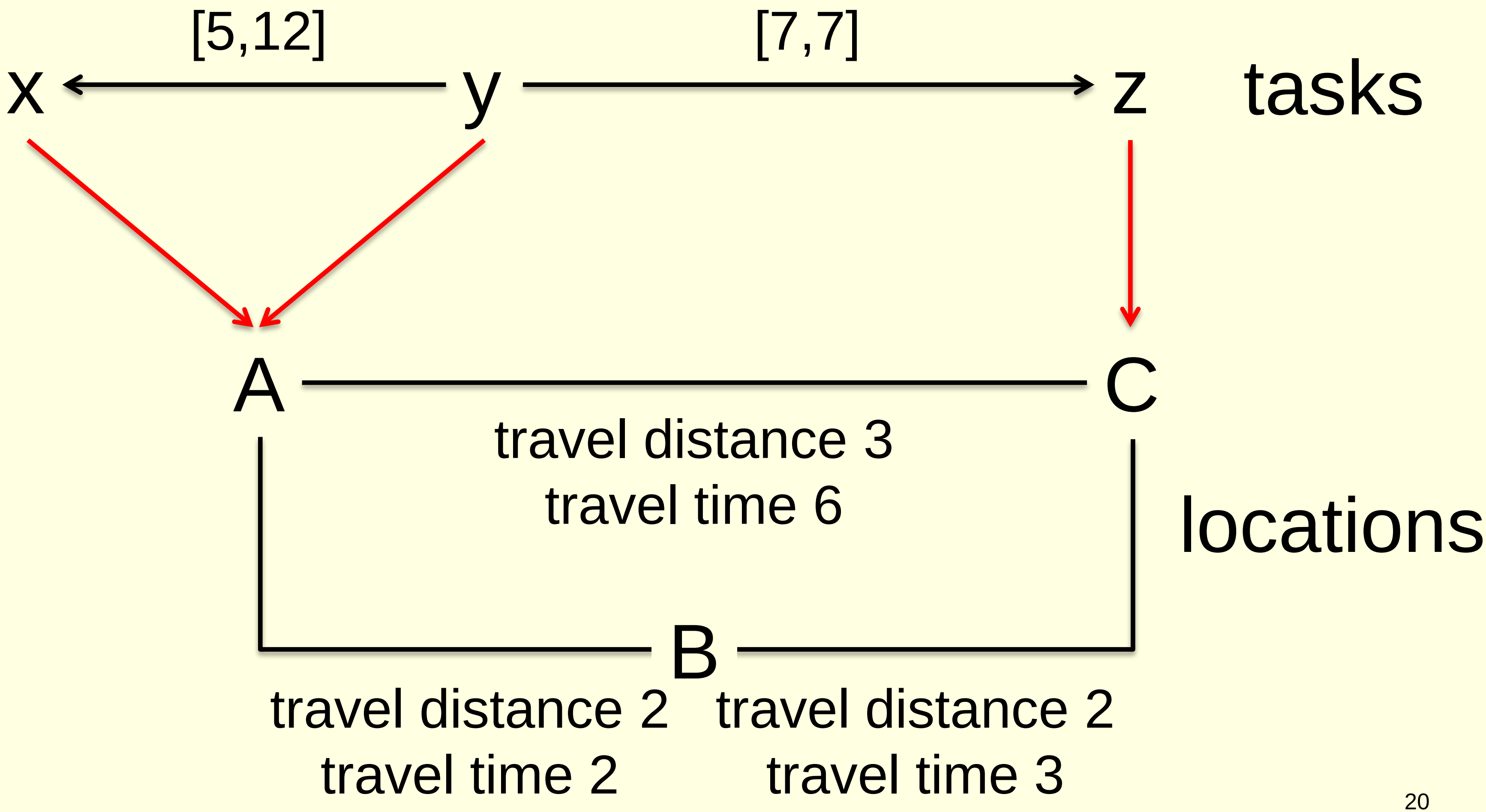
  - TSP: optimization (objective function)
    - V: locations
    - d: travel distances between locations
  - STP: satisfaction (constraints)
    - X: tasks
    - E: STCs between tasks $a \xrightarrow{[m,n]} b$
  - Connecting the STP to the TSP
    - c: mapping of tasks to locations
    - t: travel times between locations

TSP-STC: Definition

- Decide on a total ordering on the tasks and an execution time for each task that satisfies the temporal constraints
- We want to find a solution of minimum cost, where the cost of a solution is the length of the induced tour

TSP-STC


[5,12]
[7,7]
x
y
z
tasks
A
C
travel distance 3
travel time 6
locations
B
travel distance 2
travel time 2
travel distance 2
travel time 3

TSP-STC: Time Complexity

Spatial Reasoning

- Traveling Salesperson Problem (TSP)

Temporal Reasoning

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Traveling Salesperson Problem with Simple Temporal Constraints (TSP-STC)

study the time complexity of solving TSP-STCs

TSP-STC: Time Complexity

- Special cases of TSP-STCs
  - TSPs
  - STPs

TSP-STC: Time Complexity

- TSP
  - Find a Hamiltonian cycle of minimum cost on an edge-weighted complete graph
  - Time Complexity:
    - NP-hard and NP-hard to approximate
- TSP with metric distances
  - Travel distances satisfy the triangle inequality
  - Agent can visit locations more than once
  - Time Complexity
    - NP-hard but polynomial to approximate

TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

Constraints	TSP	ATSP
None	3/2 (path only: 5/3)	O(log |V|) (path only: O(log |V|))
	[Christofides 1976] [Hoogeveen 1991]	[Frieze et al. 1982] [Chekuri and Pál 2007]

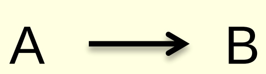
↑

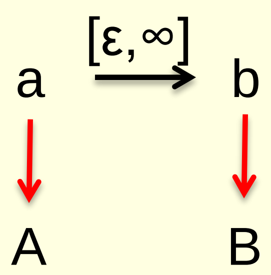
path
from a given start location
to given goal location

TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

Constraints	**TSP**	**ATSP**
None	3/2 (path only: 5/3)	O(log |V|) (path only: O(log |V|))
Path Constraints	3	O(log |V|)
Precedence Constraints	inapproximable	inapproximable





TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

Constraints	**TSP**	**ATSP**
Path Constraints	3	O(log |V|)

[Bachrach et al. 2005] [Chekuri and Pál 2007]

↑

total ordering
over a given subset of locations

A ⟶ B ⟶ C ⟶ D E F G

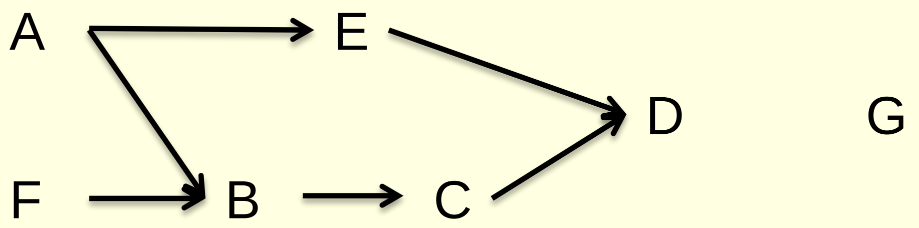
TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

Constraints	TSP	ATSP
Precedence Constraints	inapproximable	inapproximable
	[Charikar et al. 1997]	[Charikar et al. 1997]

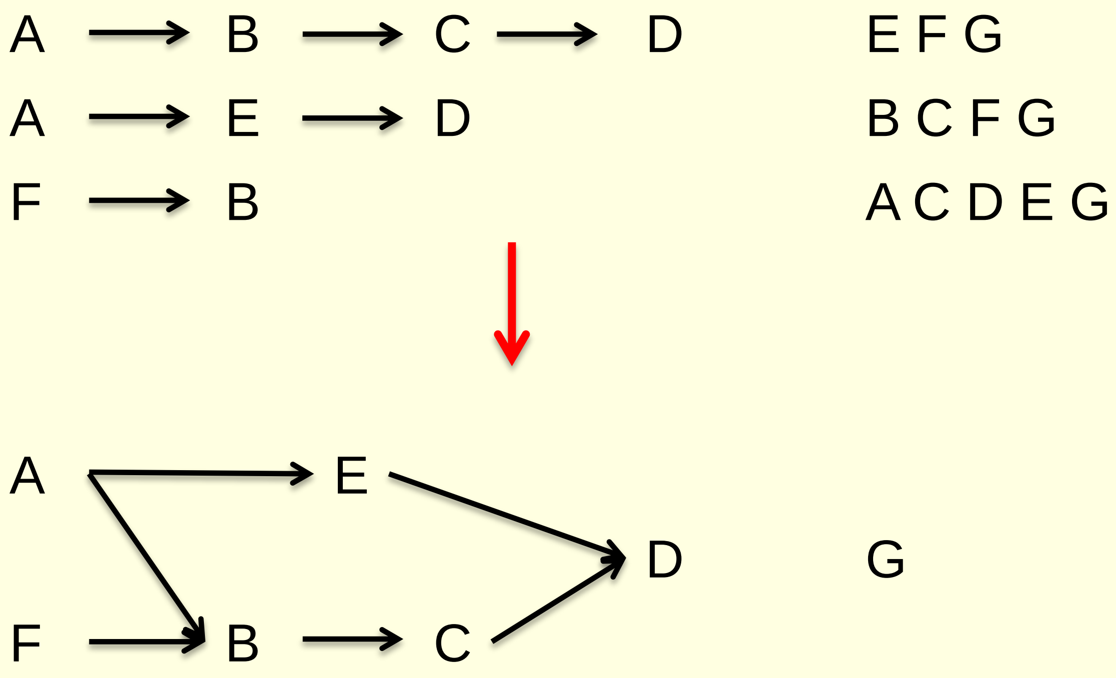
↑

partial ordering
over a given subset of locations



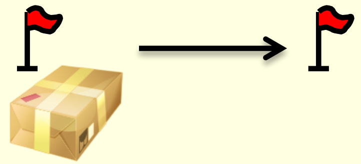
TSP-STC: Special Cases

- TSPs with Precedence Constraints generalize TSPs with Path Constraints



TSP-STC: Special Cases

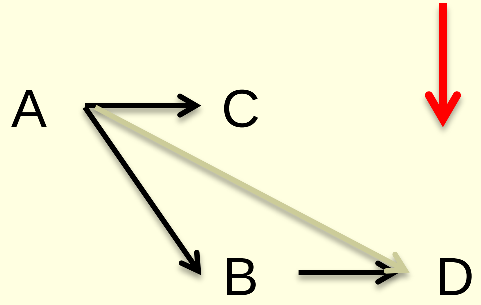
- Task a before Task b



- Task a during time window [3,8]

all travel times are zero (infinite speed)
time windows for a given subset of locations

A [1,2]   B [3,7]   C [6,9]   D[8,9]   E   F G



E   F G

TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

Constraints	TSP	ATSP
Precedence Constraints	inapproximable	inapproximable
	[Charikar et al. 1997]	[Charikar et al. 1997]

- For TSPs with precedence constraints and fixed $|V|$, there is no polynomial-time $|V|^{\alpha}$-approximation algorithm, for some $\alpha>0$, unless P=NP.
- For TSPs with precedence constraints, there is no polynomial-time $(\log |V|)^{\delta}$-approximation algorithm, for any $\delta>0$, unless $NP \subseteq DTIME(|V|^{\log \log |V|})$.

TSP-STC: Special Cases

- Time Complexity:
  existing results for metric travel distances

no constraints

many constraints

Constraints	TSP	
None	3/2 (path only: 5/3)	O(log |V|) (p
Path Constraints	3	O
Precedence Constraints	inapproximable	inap
TSP-STC	inapproximable	inap

- For TSP-STCs with fixed |V|, there is no polynomial-time $|V|^{\alpha}$-approximation algorithm, for some α>0, unless P=NP.
- For TSP-STCs, there is no polynomial-time $(\log |V|)^{\delta}$-approximation algorithm, for any δ>0, unless NP⊆DTIME($|V|^{\log \log |V|}$).

TSP-STC: General Case

Theorem

- TSP-STCs (and ATSP-STCs) are NP-hard to solve optimally and NP-hard to approximate within any polynomial factor, even with metric and symmetric travel distances and times.
- Note: TSPs with metric travel distances can be approximated in polynomial time.

TSP-STC: General Case

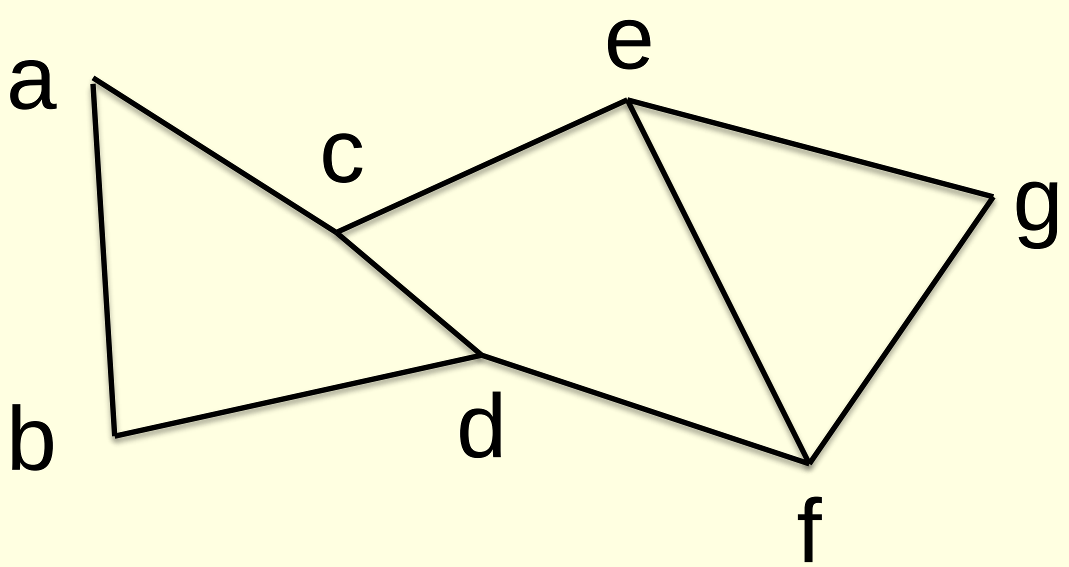
Proof Sketch

- TSP-STC
  - TSP: optimization (objective function)
  - STP: satisfaction (constraints) $\leftarrow$ NP-hard

TSP-STC: General Case

Proof Sketch

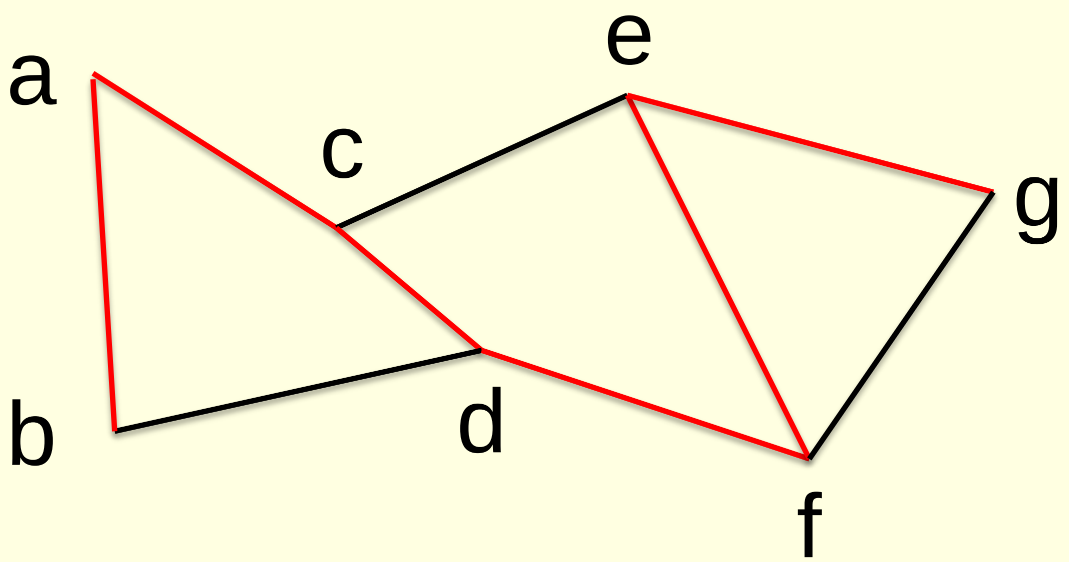
- Take any Hamiltonian path problem on an undirected graph



TSP-STC: General Case

Proof Sketch

- Take any Hamiltonian path problem on an undirected graph



TSP-STC: General Case

Proof Sketch

- Transform it into a TSP-STC with metric and symmetric travel distances and times
  - Give existing edges travel distances and times 1.0
  - Add edges to make the graph complete
  - Give new edges travel distances and times 1.5
  - Associate one unique task with each location
  - Add STCs between all pairs of tasks $a \xrightarrow{[-\infty,|V|-1]} b$

Conclusions

Constraints	TSP	ATSP
None	3/2 (path only: 5/3)	O(log |V|) (path only: O(log |V|))
Path Constraints	3	O(log |V|)
Precedence Constraints	inapproximable	inapproximable
TSP-STC	inapproximable	inapproximable

Future Work

- In the future, we would like to
  - prove that additional special cases of TSP-STCs can be solved approximately in polynomial time, especially ones that are realistic for robot applications
  - develop efficient and effective heuristics for solving TSP-STCs, based on TSP heuristics
  - study ways of combining spatial and temporal reasoning different from TSP-STCs
  - integrate the results into auction-based robot coordination systems

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  - an ONR MURI
  - the Swedish Knowledge Foundation
- The views and conclusions are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the sponsoring organizations, agencies or the U.S. government.